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Forced Korteweg-de- Vries (FKdV) Equation for spin-1/2 magnerohydrodynamic quatum plasma in presence of external periodic perturbation and extense magnetic field

Dr. Tushar Kanti Das

Assistant Professor, Dept. of Mathematics, Panchmura Mahavidyalaya, Bankura, West Bengal, India

Sourav Choudhury

KDSHM Institution, Birbhum, West Bengal, India

Abstract:

The small amplitude analysis in spin-1/2 quantum plasma, in the presence of external periodic perturbation, is studied in the framework of the quantum magneto hydrodynamic model. The Reductive Perturbation Technique (RPT) is applied to obtain the Forced Korteweg de Vries (FKdV) equation. The different plasma parameters including spit term are found to play significant role in determining the nature of the nonlinear waves the impact of the various strength of magnetic field including magnetization also studied.

Keywords: Quantum plasma, Spin 1/2 Magnetic field, RPT, FKdV equation, Magnetisation, Bohm potential.

Introduction: Currently, spin-1/2 plasma has a significant interest in the collective effects in quantum plasma. The effects of magnetic field in quantum particle give a natural elongation to the classical magneto hydrodynamic (MHD) theory. Marklund and Brodin [1] first theoretically proposed the one fluid QMHD equation which includes the effect of electron spin. This plasma occurs especially in low temperature and high density situations. So this field of physics i.e., the spin 1/2 quantum plasmas is receiving a great deal of interest in recent years. Many theoretical and experimental observations on this subject can be found in the literature, such as studies in condensed matter physics, electronics [2]. astro physics and some astronomical bodies like the interior of Jupiter, white dwarfs and super dense neutron stars [3], nano-physics, quantum-wells, quantum-wires, and quantum-dots, [4] ultracold plasmas [5], metal nanotubes and quantum diodes [6], various nonlinear optical system and laser produced plasma such as semiconductor quantum plasmas [7-9] etc. Sahu et al. [10] derived KdVB equation to study the various nonlinear parameters in spin 1/2 quantum plasma. Choudhury et al. [11] derived a two dimensional model and studied linear analysis of spin half quantum waves. Roy et al. [12] derived the couple KdV equations in spin half quantum plasma and studied the nonlinear solitary waves using the various plasma parameters. Ghorui et al. [13] also observed the phase shift relation in spin half quantum plasma during collision. Recently Jan et al.[14] studied the Fermionic spin-1/2 quantum plasma and they obtained a set of ZK equations for polarized nonlinear Alfven wave Asenjo et al. [15] investigated the relativistic corrections to the Pauli Hamiltonian in the context of spin-1/2 quantum plasma. Zhu [16] theoretically investigated the surface waves on the magnetized degenerate electron plasma half space with spin effects. Mushtaq et al [17] also studied the spin 1/2 magneto hydrodynamic model and used the tank method and obtained the KPB equation. The solution of the equation showed a general shock wave profile of solitary wave Ayub et al. [18] observed the spin evolution of electrostatic energy flow in quantum plasma. In some physical situation [19- 21] the effect of external periodic force is present. Sen et al. [22] obtained the FKdV equation from a simple plasma model. They used the forcing term assure charge density scenario. All et al observed an analytical solution for solitary waves in the presence of periodic force. Recently Ghouri et al. [24] studied the analytical solution of EASW in quantum plasma and derived the FKdV equation in presence of external periodic perturbation. However, many basic problems of the spin-1/2 quantum plasma in the presence of external periodic force are still the subject of intense experimental and theoretical studies. To the best of our knowledge there is no investigation to show the effect of external periodic force in the framework of FKdV equation in spin-1/2 quantum plasma. This is the motivation for following up this work. The organization of the present work is as follows: we introduce the model equation in section 2. In section 3 linear analyses have discussed. In section 4 we have obtained the FKdV equation using the RPT and in section 5 we have obtained numerical simulation for different values of the parameters and present discussion. The conclusion have presented in Sec.6.

Model equations: In this manuscript, we consider the spin-1/2 non relativistic degenerate electrons in quantum plasma with the effect of external periodic perturbation. The external magnetic field ʘ and the magnetization $\overrightarrow{M}$ are taken along the z direction and the form of the magnetic field and the magnetization are ∅

$\hat{z}\phi(x,t)$ and $\vec{M} = \hat{z}M(x,t)$, where $\hat{z}$ is the unit vector along the z-axis. Now we do the scaling and got,

$$r = \frac{r\Omega_i}{V_A} \quad , \quad v_i = \frac{v_i}{V_A} \quad , \quad \phi = \frac{\phi}{\phi_0} \quad , \quad t = \Omega_i t \quad , \quad n = \frac{n}{n_0} \tag{1}$$

and the velocity is taken along the x direction $\vec{v_i} = v_i(x,t)\hat{x}$, then the non linerar governing equations are [10]

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv_i) = 0 \tag{2}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{\phi}{n}\frac{\partial \phi}{\partial x} - \beta\frac{\partial n^{2/3}}{\partial x} + \frac{H_e^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n}}\frac{\partial^2\sqrt{n}}{\partial x^2}\right) + \varepsilon_0^2\beta B\frac{\partial \phi}{\partial x} + \frac{\phi}{n}\frac{\partial M}{\partial x} \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(\phi v_i) = 0 \tag{4}$$

where the various physical parameters have the following meanings :

$\beta = \frac{c_{qs}^2}{V_A^2} = \frac{2\mu_0 n_0 \varepsilon_{Fe}}{\phi_0^2}$ measures the quantum statistical effects, n_0 being the unperturbed density,

$c_{qs} = \left(\frac{2\varepsilon_{Fe}}{m_i}\right)^{1/2}$ is the quantum ion sound speed,

$H_e = \frac{\hbar\Omega_i}{\sqrt{m_e m_i}\, V_A^2}$ is a quantum parameter,

$\Omega_i = \frac{e\phi_0}{m_i}$ is the ion gyrofrequency,

$V_A = \frac{\phi_0}{(\mu_0 n_0 m_i)^{1/2}}$ is the Alfvén speed,

$\vec{M} = \varepsilon_0^2 \beta n \phi \hat{z}$ is the magnetization density,

$\varepsilon_0 = \frac{\mu_\phi \phi_0}{\varepsilon_{Fe}}$ is the Zeeman energy,

$\varepsilon_{Fe} = \frac{(3\pi^2 n_e)^{2/3}\hbar^2}{2m_e}$ is the Fermi energy of degenerate electrons,

$\mu_B = \frac{e\hbar}{2m_e}$ is the Bohr magneton and

Derivation of Forced KdV equation:

To derive the KdV equation we have used the standard RPT and the independent variables are stretched as,

$$\xi = \epsilon^{1/2}(x - \lambda t) \qquad , \qquad \tau = \epsilon^{3/2} t \tag{5}$$

where λ is the wave velocity normalized by Alfveén speed A, and is measuring the strength of nonlinearity. The dynamic variables are expanded as

$$\begin{aligned} n &= 1 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \cdots \\ \phi &= 1 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots \\ v_i &= \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \epsilon^3 v^{(3)} + \cdots \end{aligned} \tag{6}$$

and collecting terms to different orders in e, we obtain certain relations. For example, to the lowest order in ∈,

$$n^{(1)} = \phi^{(1)} = v^{(1)}/\lambda \tag{7}$$

To the first order in ∈,

$$n^{(2)} = \phi^{(2)} \tag{8}$$

and so on. After straightforward algebra gives

$$\lambda^2 = 1 + \frac{2}{3}\beta \tag{9}$$

and the final eq. is obtained as the Korteweg-de Vries (KdV) equation:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + P\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + Q\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{10}$$

where the nonlinearity coefficient P, dispersion coefficient Q.

$$P = \frac{1}{2\lambda}\left(3 + \frac{16}{9}\beta - 12\varepsilon_0^2\beta\right), Q = -\frac{H_e^2}{8\lambda}, \tag{11}$$

Now in presence of external periodic force in the system i.e. $f_o cos(wt)$, the Eqs. (14) takes the form

$$\frac{\partial \phi_1}{\partial \tau} + P\phi_1\frac{\partial \phi_1}{\partial \xi} + Q\frac{\partial^3 \phi_1}{\partial \xi^3} \quad = f_0 cos(wt) \tag{12}$$

here f_o is the peak value of the external periodic force and w is the frequency of perturbation. The equation (16) is known as the FKdV equation. If $f_o = 0$ then Eqs. (16) becomes the well known KdV equation and it's solution is given bellow,

$$\phi = \phi_m sech^2\left(\frac{\xi - \lambda_0\tau}{W}\right) \tag{13}$$

where $\emptyset_m = \emptyset m = \frac{3}{2a}$is the amplitude and $W = 2\sqrt{\frac{Q}{\lambda 0}}$ is the width of the wave, λ_0 is the normalized con- stant speed of the wave frame. Now we consider the momentum conservation law which corresponds
to

$$I = \int_{-\infty}^{+\infty} \phi^2 dx \tag{14}$$

is conserved for FKdV equation. Then the slow time dependent solution of Eq.(16) can be assumed of the form,

$$\phi_1 = \phi_m(\tau) sech^2(\frac{\xi - \lambda(\tau)\tau}{W(\tau)}) \qquad (15)$$

where, amplitude $\emptyset m(\tau) = \sqrt{\frac{3\lambda(\tau)}{P}}$ width $W(\tau) = 2\sqrt{\frac{Q}{\lambda(\tau)}}$ and velocity $\lambda(\tau)$ have to be determined. After some mathematical algebra we obtained,

$$I = \frac{24\sqrt{Q}}{P^2}\lambda^{\frac{3}{2}}(\tau) \qquad (16)$$

From Eqs (18) and (20), the value of $\lambda(\tau))$ is calculated as,

$$\lambda(\tau) = \lambda + \frac{2Pf_0}{3W} sin(w\tau) \qquad (17)$$

So the analytical solution of of spin-1/2 quantum plasma waves for the FKdV Equation (16) is,

$$\phi = \phi_m sech^2\left(\frac{\xi - \lambda(\tau)\tau}{\lambda(\tau)}\right) \qquad (18)$$

here $\phi_m(\tau) = \frac{3\lambda(\tau)}{P}$ and $W(\tau) = 2\sqrt{\frac{Q}{\lambda(\tau)}}$.

Numerical Results and Discussions: Now to check the propagation of small amplitude spin-1/2 solitary wave, we study the nonlinear characteristics of magnetosonic structure through the solution of the FKdV equation(16), for vari- ous physical plasma parameters. Figs.(1-9)reveal the propagation and interaction of nonlinear wave structures for several values of the magnetic diffusivity parameter, quantum diffraction parame ter He, Zeeman energy én, the quantum statistical parameter, fermi energy of degenerate electrons, magnetisation, frequency and strength of the external periodic perturbation respectively for small amplitude. In Fig. 1 we have plotted the solitary wave profile of one soliton solution of forced KdV equation (16) for several values of with H,-0.10, -0.50, -0.10, M-2.10, w=0.56, A-2.0, V=1.10 and τ =1. We observed that the amplitude of solitary wave profile of spin 1/2 quantum plasma waves decreases when the strength of the external periodic force decreases. In Fig.2 we have plotted the solitary wave profile of one soliton solution of forced KdV equation (16) for several values of collective electron tunneling parameter

H_e=0.20, *E*=0.30, 1.20, -0.55,8-0.15, M-2.10, w=0.75, A-2.5, V-1.12 and 7=1. It is seen that when quantum parameter H increases, the amplitude of spin 1/2 quantum plasma as well as the width of the solitary wave decreases. In Fig.3 we have plotted the variation of the solitary wave solution of the FKdV equation for different values of the frequency(w) of the external periodic force with the other parameters are fo=0.50, -0.60, 3=0.10, M-2, A-2.0, V=1.15,H=0.15 and 7=1. It is critically observed that when w= 2.80 then the amplitude of solitary wave is large and when w = 0.10 then the amplitude of solitary wave is small. So it is found that when w increases then the amplitude and width of the spin 1/2 quantum plasma waves increases. In Fig. 4, we have plotted the variation of the solitary wave solution of the FKdV (Eq. (16)) for different values of A with other parameters are e=0.50, 8-0.15, M-2.50, w=0.75, V-1.10,H,-0.10 and 71. It is observed that when the parameter of A increases amplitude of spin half solitary wave increases, but the width of the solitary wave decreases. The variation of the amplitude of the solitary wave solution w.r.t. the various values of frequency of the external periodic force is depicted in Fig. 5 and other parameters are same as in Fig. 4. It is critically observed that when w = 0.10, spin half quantum solitary waves has larger amplitude than w = 0.80 and w = 1.50. So it is found that when w increases then the amplitude of the solitary wave decreases rapidly. In Fig. 6, the variation of amplitude of the spin half solitary wave solution of the forced KdV (Eq. (16)) with respect to fo is presented for different values of A and the other parameters are same as in Fig. 5. Here we noticed that when A =0.40, the solitary waves has larger amplitude but when λ= 1.50 then it has smallest amplitude comparable to the values of A = 0.80 and 0.40. So when A increases, the amplitude of spin half solitary wave decreases rapidly. In Fig.7, we have plotted the variation in the width of the spin half solitary wave solution with respect fo of the forced KdV (Eq. (16)) is presented for different parameters of w, and other parameters are c=0.60, 8-0.20, M-2.20, V-1.10,77,-0.30, A-3.0 and 7-1. It is noticed that as the parameter w increases, the width of the spin half solitary waves increases rapidly. In Fig. 8, the variation in the width of the solitary wave solution with respect fo of the forced KdV (Eq. (16))is presented for different values of quantum diffraction and the other parameters are same as in Fig. 7. We have noticed that when the quantum tunneling parameter II increases the width of the solitary waves increases rapidly. In Fig. 9, the variation in the width of plasma wave solution with respect to fo of the forced KdV (Eq. (16)) is presented for different values of X and the other parameters are fixed as in Fig.8. It is observed that after crossing the value fo= 0.75, then the width of the spin half solitary waves (A = 0.50) profile decrease rapidly than A= 0.60 and X = 0.70 and at the point fo= 0.90 the solitary wave decreases rather than the value of = 0.70. So we have seen that when A increases, the width of the solitary waves increases rapidly. So, the FKdV equation gives a good estimate of the non linear structures in spin half plasma waves, in the presence of external periodic force.

Conclusions: As a summary, we carried out the analytical solitary wave solution in presence of external force of spin plasma for FKdV equation. We assumed a plasma model and carried out a numerical analysis of plasma waves. To check how the different parameters affect the quantum plasma waves, we plotted number of graphs. We applied the RPT to obtain FKdV equation. Finally, the results obtained here may be useful to study the nonlinearities of spin plasma in presence of external perturbation.

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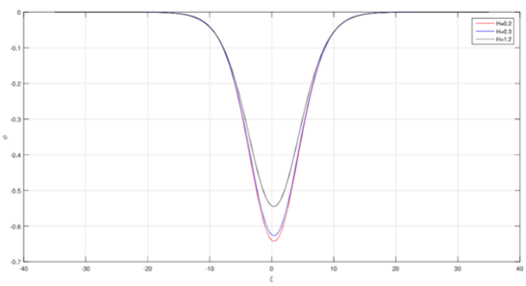

Figure 1: **The solitary wave profile of one soliton solution of forced KdV equation (16)for several values of f_0 with H_e=0.10, ϵ=0.50, β=0.10, M=2.10, w=0.56, λ=2.0, V=1.10 and** τ=1.

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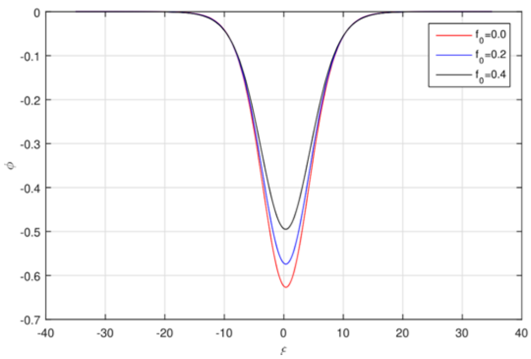

Figure 2: **The solitary wave profile of one soliton solution of forced KdV equation (16)for several values of H_e with f_0=0.40, ϵ=0.55, β=0.15, M=2.10, w=0.75, λ=2.5, V=1.12 and** τ=1.

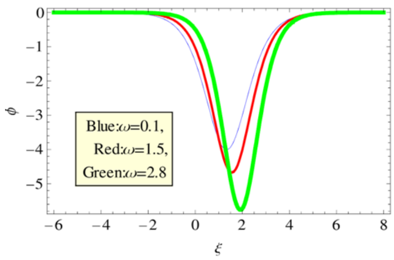

Figure 3: **The solitary wave profile of one soliton solution of forced KdV equation (16)for several values of w with f_0=0.50, ϵ=0.60, β=0.10, M=2, λ=2.0, V=1.15,H_e=0.15 and** τ=1 .

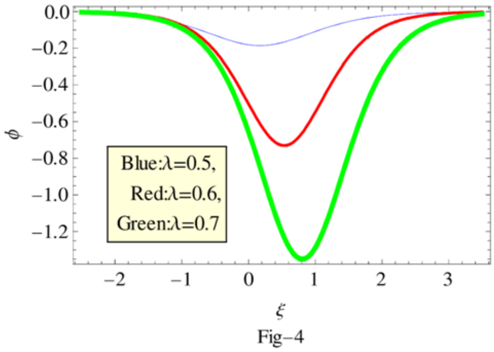

Figure 4: **The solitary wave profile of one soliton solution of forced KdV equation (16)for several values of λ with ϵ=0.50, β=0.15, M=2.50, w=0.75, V=1.10,H_e=0.10 and** τ=1.

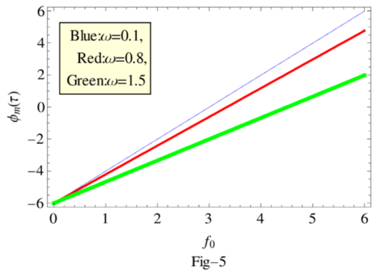

Figure 5: **Variation of amplitude of one soliton solution of forced KdV equation (16)for several values of w with λ=2.0, ϵ=0.50, β=0.15, M=2.50, V=1.10,H_e=0.10 and** τ=1.

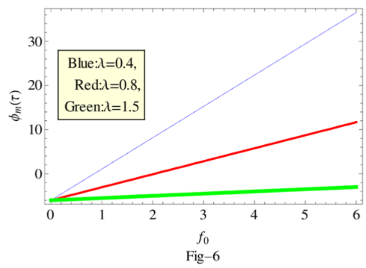

Figure 6: **Variation in the amplitude of one soliton solution of the forced KdV equation (16) for different values of λ with respect to f_0 with ϵ=0.50, β=0.10, M=2.10, V=1.10,H_e=0.10,w=0.56 and** τ=1 .

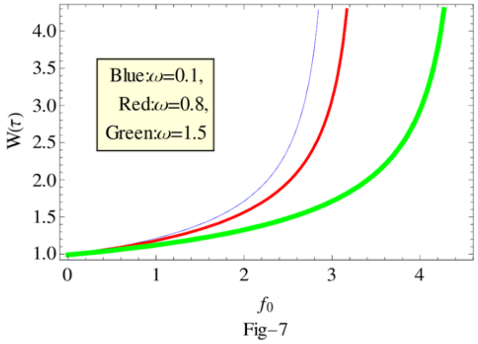

Figure 7: **Variation in the width of one soliton solution of the forced KdV equation (16) for different values of w with respect to f_0 for ϵ=0.60, β=0.20, M=2.20, V=1.10,H_e=0.30, λ=3.0 and τ=1.**

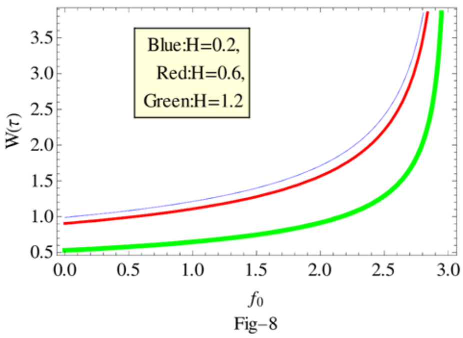

Figure 8: **Variation in the width of one soliton solution of the forced KdV equation (16) for different values of H_e with respect to f_0 for ϵ=0.60, β=0.20, M=2.20, V=1.10,w=0.56, λ=3.0 and τ=1.**

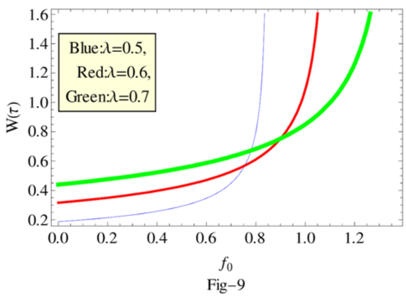

Figure 9: **Variation in the width of one soliton solution of the forced KdV equation (16) for different values of λ with respect to f_0 for ϵ=0.60, β=0.20, M=2.20, V=1.10,w=0.56, H_e=0.10 and τ=1.**