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## **A Study on Sponge Iron Based Transportation Production and Pollution under General Supply Chain Model**

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### **Abstract:**

*Over the last few years, sponge iron-based production and transportation problems for major sponge iron producing countries are triggering a critical issue. The excess of marginal pollution from production industries and their disintegration take drives towards the change of policymaking. The sustainable development of any country signifies the reduction of biohazards, which in turn improves the health index and livelihood status of people across the world. In this study, we have shown how production, rail freight transport relates to pollution. To draw several graphs and numerical computations we use MATLAB software and LINGO software respectively. The comparative study has been presented using general fuzzy systems. Lastly, we have justified our proposed model using sensitivity analysis along with graphical interpretation.*

**Keywords: Production; Pollution; Transportation; General Fuzzy system; Modelling; Optimization.**

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**Introduction:** Due to globalization, the manufacturing of iron as well as steel and other materials, such as aluminium and materials for chemical-based products are playing a substantial role in the environment that we have created. This impact is related to total energy use and different levels of pollution came from various stages in the production process. The ferroalloys production is minor in contrast with base materials such as steel and aluminium. The major portion of complete man-made pollution has been incorporated by the severe environmental effects of silicon and ferroalloys.

In the literature, several research articles are available in which most of them are associated to cost benefits and controlling carbon emissions from the vehicles used in the transportation itself. Sarkar et al. (2015) studied the effect of unequal lot size model for variable set up cost and carbon emission cost in a supply chain (SC) problem. Moreover, to study the problem when some of the parameters are becoming non-randomly uncertain fuzzy system may be utilized. After the development of fuzzy set theory by Zadeh (1965), several attempts have been made along these directions by eminent thinkers [De et al. (2014), De and Sana (2013), Kumar et al.(2012)] Beyond this, the triangular dense fuzzy

set (TDFS) [De and Beg (2017)] and cloudy fuzzy set [De and Mahata (2016)], Karmakar et al. (2017, 2018)] have kept a special destination to tackle the problem of the learning effect in any production process extensively. Hence in our study we present out article that includes cost minimization two-layer SC problem having two-way pollution channel under learning fuzzy environment.

**Preliminaries:**

**Choice of Pollution function** [Karmakar et al. (2017)]: Let  $X \equiv X(t)$  be the production rate and  $Y \equiv Y(t)$  be the pollution rate. Then, for the absence of environmental pollution, the production rate will decrease exponentially with the maximum consuming capacity of the customers. The production rate will also decrease slowly due to the unavailability of raw materials from the decaying environmental resources or pollutant environment. However, in the absence of production, the pollution rate decreases proportionally and increases exponentially with the presence of production itself. Thus, the governing differential equation of production-pollution rate can be defined as follows:

$$\begin{cases} \dot{X} = aX - rX^2 - \alpha XY, & a, r, \alpha > 0 \\ \dot{Y} = -cY + \gamma XY, & c, \gamma > 0 \end{cases} \quad (1)$$

Now, solving the differential equation (1) with the proper choice of  $a, r, \alpha, c$  and  $\gamma$  we get the functional dependence of amount of pollution  $Y$  (%) and production rate  $p$  metric tonnes (MT) per time under pollution control measures (PCM) and it is given by

$$y = 0.45 + 0.01p - 0.25 \log \log (p) \quad (2)$$

**Normalized General Triangular Fuzzy Number (NGTFN):** Let  $A$  be a NGTFN having the form  $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ . Then the membership function of the fuzzy set  $\tilde{A}$  is defined by

$$\mu(\tilde{A}) = \begin{cases} 0, & \text{if } a < a_1 \text{ and } a > a_3 \\ \frac{a-a_1}{a_2-a_1}, & \text{if } a_1 \leq a \leq a_2 \\ \frac{a_3-a}{a_3-a_2}, & \text{if } a_2 \leq a \leq a_3 \end{cases} \quad (3)$$

Now, the index value of  $\mu(\tilde{A})$  [due to Yager (1981)] is obtained as

$$I(\tilde{A}) = \frac{1}{2} \int_0^1 [L(\alpha) + R(\alpha)] d\alpha = \frac{(a_1 + 2a_2 + a_3)}{4} \quad (4)$$

For the left and right  $\alpha$ -cuts  $L(\alpha) = a_1 + (a_2 - a_1)\alpha$  and  $R(\alpha) = a_3 - (a_3 - a_2)\alpha$  respectively

**Notations and Assumptions:**

**Notations:**

1.  $p$  : Production rate per cycle (MT/year) (Decision variable)
2.  $y$  : Pollution index (%)
3.  $\tau_1$  : Production run time (Decision variable) (year)
4.  $\tau_2$  : Transportation time (year)
5.  $\tau_3$  : Inventory exhausts time (year)

- 6.  $d$ : Demand rate per cycle (MT/year)
- 7.  $q$ : Order quantity (MT)
- 8.  $\delta$ : Deterioration rate per unit time
- 9.  $l$ : Transportation distance (Mile)
- 10.  $C_p$ : Production cost per unit item (\$)
- 11.  $h_p$ : Holding cost per item per unit time at producer’s plant (\$)
- 12.  $h_r$ : Holding cost per item per unit time at retailer’s shop (\$)
- 13.  $C_{pol}$ : Pollution cost (\$) (per unit item)
- 14.  $C_t$ : Transportation cost (\$) (per unit MT per Mile)
- 15.  $C_d$ : Deterioration cost (\$) (per unit item per unit time)
- 16.  $C_c$ : Global social cost of carbon (\$)
- 17.  $k_1$ : Setup cost at production plant (\$)
- 18.  $k_2$ : Setup cost at retailer side (\$)

**Assumptions:**

- 1. Shortages are not allowed.
- 2. Lead time is zero.
- 3. Producer has the sole responsibility to transport the items to retailer.
- 4. Deterioration occurs and deteriorated items cannot be recoverable.

**Case Study:** Let us extend the case study performed by Karmakar et al. (2017, 2018). These studies were basically involved to the manufacturing and pollution of a sponge iron industry. Our focus of interest is to measure pollution due to transportation of products by freight train and simultaneously to minimize the average inventory cost via managerial insights and learning experiences. This industry has single managerial system by encircling the industry with radius of 600 km (approximately). The orders are placed instantly by the separate transporting system. The data information like set up cost, holding cost, production cost, transportation cost, pollution cost, carbon emission cost obtained from the industry is given in Table 1.

Table 1: Observed data set for the proposed industry

Set-up cost per cycle \$20000	Holding cost per cycle in production plant per MT \$5	Deterioration cost per cycle in production plant per unit item \$10	Deterioration fraction 0.1
Pollution cost due to production per MT \$43.89	Holding cost per cycle in retailer plant per MT \$10	Production cost per MT \$327.56	
Social cost of carbon per MT \$417	Transportation cost per gallon fuel \$3.5	Length covered by freight train 600 Miles	

**The research problems are:**

1. Whether the proposed SI industry could control the integrated pollution of the supply chain by reaching annual average cost minimum.
2. What is the optimum order quantity for which the average inventory cost is getting minimum?
3. Whether the general fuzzy system is more effective to reduce the pollution of the supply chain as well as average inventory cost than the crisp system.

**Formulation of Supply Chain (SC) Model:** Let the production process starts with zero inventories with production rate  $p$  with deterioration fraction  $\delta$ . During the production run time  $\tau_1$ , the inventory level gradually increases and it reaches its maximum value at the end of time  $\tau_1$ . Then finished product is transported to the retailer at time span  $(\tau_2 - \tau_1)$ . During transportation, the production begins with zero stock. At the time  $\tau_2$ , the items are received and began to sale at the retailer’s counter and the inventory will get at time  $\tau_3$  because of demand  $d$ . During the interval  $[\tau_2, \tau_3]$ , the variation in the inventory depletes for demand only. Therefore, the governing differential equations of the model (shown in Figure 1) are obtained as follows:

$$\frac{dq_1(t)}{dt} = p - \delta q_1(t), \quad 0 \leq t \leq \tau, \quad q_1(0) = 0$$

(8)

$$\frac{dq_2(t)}{dt} = -d, \quad \tau_2 \leq t \leq \tau_3, \quad q_2(\tau_3) = 0$$

(9)

Solving (8) we get,

$$q_1(t) = \frac{p}{\delta} (1 - e^{-\delta t}), \quad 0 \leq t \leq \tau_1$$

(10)

And that for (9) we have,

$$q_2(t) = d(\tau_3 - t), \quad \tau_2 \leq t \leq \tau_3$$

(11)

Also, for continuation of (10) and (11) we write

$$q = \frac{p}{\delta} (1 - e^{-\delta \tau_1}) = d(\tau_3 - \tau_2)$$

(12)

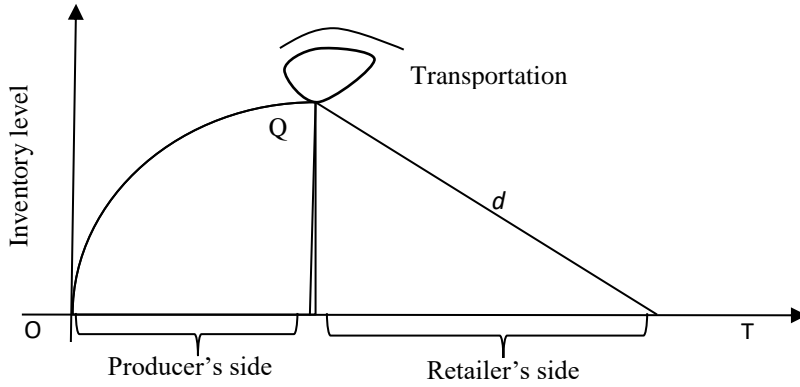


Figure 1: Production transportation SC model

Now, the inventory holding cost (HC) for the SC is given by

$$\begin{aligned}
 HC &= h_p \int_0^{\tau_1} q_1(t)dt + h_r \int_{\tau_2}^{\tau_3} q_2(t)dt \\
 &= \frac{h_p p}{\delta} \left[ \tau_1 + \frac{e^{-\delta\tau_1} - 1}{\delta} \right] + \frac{h_r d}{2} (\tau_3 - \tau_2)^2
 \end{aligned}
 \tag{13}$$

Then the production cost (PC) is given by

$$PC = C_p \left[ \int_0^{\tau_1} q_1(t)dt \right] = \frac{C_p p}{\delta} \left[ \tau_1 + \frac{e^{-\delta\tau_1} - 1}{\delta} \right]
 \tag{14}$$

Also the deterioration cost

$$DC = C_d (p\tau_1 - q) = C_d p \left[ \tau_1 - \frac{1 - e^{-\delta\tau_1}}{\delta} \right]
 \tag{15}$$

The transportation cost (TC) is given by

$$TC = C_t \times \frac{2lq}{471} = 0.00424628lqC_t
 \tag{16}$$

Pollution cost due to transportation (TPC) is given by

$$TPC = C_c \times \frac{2lq}{471} \times 22.38 \times 0.000454 = 0.0000431445lqC_c
 \tag{17}$$

Pollution cost due to production (PPC) is given by

$$PPC = C_{pol} \times p\tau_1
 \tag{18}$$

Joint set-up cost

$$SC = k_1 + k_2 \tag{19}$$

**Time Management of the production process:** Time management is an essential factor for any kind of transportation. Since the freight train travels  $2l$  distance during time interval  $(\tau_2 - \tau_1)$  (shown in Figure 1) and at the time  $(\tau_3 - \tau_2)$  retailer’s inventory reaches zero and hence best possible time to reach the items again at time is  $\tau_2$ . Thus, we have

$$\tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1 \tag{20}$$

Thus, the total average joint inventory system (SC) cost ( $z$ ) is given by

$$z = \frac{1}{\tau_1} [HC + PC + DC + TC + TPC + SC + PPC]$$

$$z = \frac{h_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + C_d p \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + \frac{C_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + C_{pol} p + \frac{K_1}{\tau_1} + C_t \times 0.00424628ld + C_c \times 0.0000431445ld + \frac{h_r d \tau_1}{2} + \frac{K_2}{\tau_1} \tag{21}$$

Finally, the problem of supply chain model is given by

$$\{Minimize z = \frac{h_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + C_d p \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + \frac{C_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + C_{pol} p + \frac{K_1}{\tau_1} + C_t \times 0.00424628ld + C_c \times 0.0000431445ld + \frac{h_r d \tau_1}{2} + \frac{K_2}{\tau_1} \text{ subject to, } \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1 \quad q = \frac{p}{\delta} (1 - e^{-\delta\tau_1}) = d\tau_1 \quad y = 0.45 + 0.01p - 0.25 \log \log (p) \tag{22}$$

**Formulation of General Fuzzy SC Model:** Let us assume all the cost parameters (denoted by the cost vector  $\underline{C}$ ) and demand rate ( $d$ ) associated with the model are flexible in nature and they might follow triangular fuzzy numbers in the following form:  $\tilde{C}_i = \langle C_{i1}, C_{i2}, C_{i3} \rangle, i = 1, 2, \dots, 9,$   $\tilde{d} = \langle d_1, d_2, d_3 \rangle$  where  $\tilde{C}_i = (C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9) = (h_p, C_d, C_p, C_{pol}, K_1, K_2, h_r, C_t, C_c)$ . Also, due to fuzzification of parameters like demand rate, the order quantity, production rate and pollution level will assume values in the following:

$$\{\tilde{q} = \langle q_1, q_2, q_3 \rangle = \langle d_1 \tau_1, d_2 \tau_1, d_3 \tau_1 \rangle \quad \tilde{p} = \langle p_1, p_2, p_3 \rangle = \langle d_1 \tau_1 \delta (1 - e^{-\delta\tau_1}), d_2 \tau_1 \delta (1 - e^{-\delta\tau_1}), d_3 \tau_1 \delta (1 - e^{-\delta\tau_1}) \rangle \quad \tilde{y} = \langle y_1, y_2, y_3 \rangle = \langle 0.45 + 0.01p_1 - 0.25 \log \log p_3, 0.45 + 0.01p_1 - 0.25 \log \log p_3, 0.45 + 0.01p_1 - 0.25 \log \log p_3 \rangle \tag{23}$$

Then the corresponding fuzzy problem of the crisp problem (22) can be written as

$$\{\tilde{z} \cong \tilde{p} \sum_{i=1}^4 \tilde{C}_i f_i + \tilde{C}_5 f_5 + \tilde{C}_6 f_6 + \tilde{d} \sum_{i=7}^9 \tilde{C}_i f_i \text{ subject to, } \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1, \\ \tilde{q} \cong \frac{\tilde{p}}{\delta}(1 - e^{-\delta\tau_1}) \cong \tilde{d}\tau_1 \tilde{y} \cong 0.45 + 0.01\tilde{p} - 0.25 \log \log \tilde{p} \quad (24)$$

Where

$$\{f_1 = \frac{1}{\delta} \left(1 + \frac{e^{-\delta\tau_1}-1}{\delta\tau_1}\right), f_2 = \left(1 + \frac{e^{-\delta\tau_1}-1}{\delta\tau_1}\right), f_3 = \frac{1}{\delta} \left(1 + \frac{e^{-\delta\tau_1}-1}{\delta\tau_1}\right), f_4 = 1, f_5 = \frac{1}{\tau_1}, f_6 = \frac{1}{\tau_1}, f_7 = \frac{\tau_1}{2}, f_8 = 0.00424628l, f_9 = 0.0000431445l \quad (25)$$

**Defuzzification of the general fuzzy model:** Since all the fuzzy parameters assume triangular fuzzy numbers so the fuzzy objective  $\tilde{z}$  must follow the triangular fuzzy number of the form  $\tilde{z} = \langle z_1, z_2, z_3 \rangle$  whose components are obtained below:

$$\{z_1 = p_1 \sum_{i=1}^4 C_{i1} f_i + C_{51} f_5 + C_{61} f_6 + d_1 \sum_{i=7}^9 C_{i1} f_i \quad z_2 = p_2 \sum_{i=1}^4 C_{i2} f_i + C_{52} f_5 + C_{62} f_6 + d_2 \sum_{i=7}^9 C_{i2} f_i \quad z_3 = p_3 \sum_{i=1}^4 C_{i3} f_i + C_{53} f_5 + C_{63} f_6 + d_3 \sum_{i=7}^9 C_{i3} f_i \quad (26)$$

Therefore, utilizing (4) the equivalent crisp problem of the given fuzzy problem (24) can be replaced by the index value of the fuzzy parameters along with given constraints is defined as

$$\text{Minimize } I(\tilde{z}) = \frac{1}{4}(z_1 + 2z_2 + z_3) \quad (27)$$

Subject to

$$\{\tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1, I(\tilde{d}) = \frac{(d_1 + 2d_2 + d_3)}{4}, I(\tilde{q}) = I(\tilde{d})\tau_1, I(\tilde{q}) = \frac{I(\tilde{p})}{\delta}(1 - e^{-\delta\tau_1}), I(\tilde{y}) = 0.45 + 0.01 I(\tilde{p}) - 0.25 \log \log [I(\tilde{p})] \quad (28)$$

And the values of  $z_i, i = 1, 2, 3$  are found from (26)

**Numerical Illustration:** Using the data set stated in Table 1 and utilizing the pollution function (2) stated at subsection 2.1 in the original SC problem we get the optimal results and they are put in Table 2. However, we compute the supply chain (SC) cost of the problem (24) for general fuzzy approach.

Table-2: Optimal solutions of the proposed SC model under several environments

Model	$p^*$ (MT)	$y^*$ (%)	$\tau_1^*$ (Year)	$\tau_2^*$ (Year)	$\tau_3^*$ (Year)	$q^*$ (MT)	$z^*$ (\$)	$\frac{Z^* - Z_*}{Z_*} \times 100\%$
Crisp	611.37	4.96	0.3768	0.5652	0.9620	226.07	117542.80	0
General Fuzzy	596.12	4.82	0.3779	0.5668	0.9447	221.06	114160.20	-2.88

Table 2 is showing the optimal SC cost with respect to optimum order quantity, cycle time, and pollution level and production rate for three different scenarios namely crisp and general fuzzy system. For the general fuzzy system, the SC cost increases to \$ 114160.20 by reducing the negligible pollution to 4.82% with order quantity 221.06 MT. But the crisp model is quite expensive with SC cost \$ 117542.80, pollution level 4.96% and order quantity 226.07 MT. The general fuzzy system it is 2.88% only with respect to the crisp optimal solution.

**Conclusions:** In this study we have developed a two-channel pollution level on two-layer supply chain deteriorated sponge iron manufacturing model under general fuzzy environment. For channel 1 of pollution corresponds the amount of item produced per time and that for second channel it depends upon the distances travelled and items transported. The production run time and the demand quantity may matter over the minimization of SC cost and pollution control. For sustainability a situation has come to balance production-demand-pollution-production run time altogether and this is only possible when the DM opts general fuzzy system.



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